***Linear Differential Equations with variables***

***Coefficients***

An equation of the form



where,  are constants and is function of *x* or constant, is called the linear differential equation with variables coefficients.

**NOTE:** If we put , then the equation (1) is transformed into an equation with constant coefficients changing the independent variable from *x* to *t* as,



Now







Again, 















Similarly, 

… … … … … … … … … … … … …



From (1) we get,



The equation (2) is a linear differential equation with constant coefficients.

**Problem-01:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,







Let, be the trial solution of the equation (2)

Then the auxiliary equation of (2) is,

















The general solution of (1) is,







where, , are arbitrary constants.

**Problem-02:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,







Let, be the trial solution of the equation (2)

Then the auxiliary equation of (2) is,











The general solution of (1) is,









where, , are arbitrary constants.

**Problem-03:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,







Let, be the trial solution of the equation (2)

Then the auxiliary equation of (2) is,















The general solution of (1) is,









where, , are arbitrary constants.

**Problem-04:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,







Let, be the trial solution of the equation (2)

Then the auxiliary equation of (2) is,























The general solution of (1) is,









where, , are arbitrary constants.

**Problem-05:**Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Therefore the general solution is,





where, , are arbitrary constants.

**Problem-06:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,











Therefore the general solution is,





where, , are arbitrary constants.

**Problem-07:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,











The complementary function of (1) is,





The particular integral of (1) is,











Therefore the general solution is,





where, , are arbitrary constants.

**Problem-08:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,























Therefore the general solution is,





where, , are arbitrary constants.

**Problem-09:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Now let, 





which is linear equation

Therefore, 





Integrating,







Again, 







which is also a linear equation

Therefore, 





Integrating,









Therefore the general solution is,





where, , are arbitrary constants.

**Problem-10:** Solve 

**Solution:** Given that,

Putting in equation (1) we get,





Let, be the trial solution of the corresponding homogeneous equation



Then the auxiliary equation of (3) is,















The complementary function of (1) is,







The particular integral of (1) is,









Let , 





which is linear equation

Therefore, 





Integrating,















Again, 





which is also a linear equation

Therefore, 





Integrating,





















Therefore the general solution is,





where, , are arbitrary constants.

**Exercise: Try Yourself:**

**01:**Solve 

**02:**Solve 

**03:**Solve 

**04:**Solve 

**05:**Solve 

**06:**Solve 